



AFGL-TR-80-0307

LEVELI

STUDY OF UPPER ATMOSPHERE WIND MOTIONS

W.B. Hanson

W.R. Coley

R.A. Heelis

Center for Space Sciences University of Texas at Dallas P.O. Box 688 Richardson, Texas 75080

DTIC ELECTE APR 1 4 1981

15 September 1980

Final Report 22 June 1979 - 30 September 1980

Approved for public release; distribution unlimited

FILE COPY

AIR FORCE GEOPHYSICS LABORATORY AIR FORCE SYSTEMS COMMAND UNITED STATES AIR FORCE HANSCOM AFB, MASSACHUSETTS 01731

81 4 14 .36

Qualified requestors may obtain additional copies from the Defense Technical Information Center. All others should apply to the National Technical Information Service.

(19 REPORT DOCUMENTA	TION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
L REPORT NUMBER	2. GOVT ACCES	SION NO.	3 RECIPIENT'S CATALOG NUMBER
sear les ex mars	AD-HO	97	733
AFGLHTR-80-0307			5 TYPE OF REPORT & PERIOD COVERE
4. THEE (and Spotters)	1944		Final Technical
STUDY OF UPPER ATMOSPHERE WIN	ID MOCTONS.	j	Report: 6/22/79 - 9/30/80
	-		S. PERFORMING ORG. REPORT NUMBER
The second secon		[14]	E1262-Ø1
3-Au Thoras			6. CONTRACT OR GRANT NUMBER(*)
W.B. Hanson)
W.R./Coley			1) (L
R.A./Heelis	1	12	F19628-79-C-Ø113
9. DES COMING ORGANIZATION NAME AND AC	DRESS		10- PROGRAM ECTION & PROJECT, TASK
Center for Space Sciences		-	AREA & WORF UNIT NUMBERS
University of Texas at Dallas	P.O. Box 688		62101F (17 1/1)
Richardson, Texas 75080	, 1.0. box 000	(16)	669004AH
	<u></u>	()	<u> </u>
11. CUNTROLLING OFFICE NAME AND ADDRES			12. REPORT DATE
Air Force Geophysics Laborato	•		15 September 1980
	01731		
Monitor/Frank Marcos/LKB	different from Controlling	()(Uca)	15. SECURITY CLASS, (of this report)
	· · ·	Omes,	Second and second
9 Final Kyt	· (Unclassified
	11 81	1 1	15. DECLASSIFICATION DOWNGRADING
du Allin I	A - A 0 1	9 !	SCHEDULE
16. DISTRIBUTION STATEMENT (of this Report)			L
Approved for public release;	distribution u	nlímít	led.
Approved for public release;	distribution u	nlímít	red.
Approved for public release; 17. DISTRIBUTION STATEMENT of the aboutact			
17. DISTRIBUTION STATEMENT of the abetract			
17. DISTRIBUTION STATEMENT of the abetract			
17. DISTRIBUTION STATEMENT of the abetract			
17. DISTRIBUTION STATEMENT of the abetract			
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES	entered in Block 20, if di	ifereni iro	m Report)
17. DISTRIBUTION STATEMENT of the abetract	entered in Block 20, if di	ifereni iro	m Report)
17. DISTRIBUTION STATEMENT of the abetract	entered in Block 20, if di	ifereni iro	m Report)
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if nece	entered in Block 20, if di	k number	m Report)
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES	entered in Block 20, if di	k number	m Report)
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if nece	entered in Block 20, if di	k number	m Report)
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if nece NEUTRAL WIND, UPPER ATMOSPHE	entered in Block 20, if display and identity by blockers. ATMOSPHERIC	k number	RER ACCELEROMETER
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if neces NEUTRAL WIND, UPPER ATMOSPHE 20. ABSTRACT (Continue on reverse side if neces	entered in Block 20, if display and identify by block RE, ATMOSPHERIC	k number	RER ACCELEROMETER
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary notes) NEUTRAL WIND, UPPER ATMOSPHE 20. ABSTRACT (Continue on reverse side if necessary) This report summarizes	entered in Block 20, if diseases and identify by block RE, ATMOSPHERIC seasy and identify by block the efforts at 1	k number	RER ACCELEROMETER
18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary notes) NEUTRAL WIND, UPPER ATMOSPHE 20. ABSTRACT (Continue on reverse side if necessary necessar	entered in Block 20, if dispersion of the efforts at the AFGL accelerome.	k number) EXPLO	RER ACCELEROMETER Las to evaluate the crosswin MESA's) mounted on the
17. DISTRIBUTION STATEMENT of the abetract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary notes) NEUTRAL WIND, UPPER ATMOSPHE 20. ABSTAT (Continue on reverse side if necessary) This teport summarizes	entered in Block 20, if dispersion of the efforts at the AFGL accelerome.	k number) EXPLO	RER ACCELEROMETER Las to evaluate the crosswin MESA's) mounted on the
17. DISTRIBUTION STATEMENT of the abstract 18. SUPPLEMENTARY NOTES 19. KEY WORDS (Continue on reverse side if necessary to continue on reverse side if necessary to the summarizes measuring capability of the	entered in Block 20, if dispersion of the second identify by block the efforts at lAFGL accelerometers. The softwar	k number) EXPLO r number) JT Dallers (RER ACCELEROMETER Las to evaluate the crosswith MESA's) mounted on the ilable for this purpose from
17. DISTRIBUTION STATEMENT of the abstract 18. SUPPLEMENTARY NOTES 19. KEY WORDS / Continue on reverse side if necess NEUTRAL WIND, UPPER ATMOSPHE 20. ABSTRACT / Continue on reverse side if necess This report summarizes measuring capability of the Atmosphere Explorer satellit	entered in Block 20, if diseases and identify by block RE, ATMOSPHERIC the efforts at I AFGL accelerometes. The softwar prevented its us producing a sur	k number) EXPLO r number) JT Dall ters (re ava e for table	RER ACCELEROMETER las to evaluate the crosswin MESA's) mounted on the ilable for this purpose from our purposes, and much of on coffware package. Esing this

DD 1 JAN 73 1473 EDITION OF THINY 55 IS DESOLETE

417001

Both AE-D and AE-E have another dovice (NATE) on board that measures the

20.

same wind component as MESA. Our limited comparisons of these measurements show that they are in reasonable agreement. We suggest that a substantial amount of MESA wind data from these two satellites be processed in order to make a careful comparison with the NATE.

On AE-C NATE does not measure the horizontal wind component, and we suggest that the MESA wind data for this satellite be processed in its entirety (for the elliptic orbit phase) and be placed in the UA file. Fortunately, the AE-C MESA data are the cleanest of all.



TABLE OF CONTENTS

I.	INTRODUCTION	•	•	•	•	•	•	•	•	•		•	•	•	2
II.	SOFTWARE	•		•		•	•	•							2
ιι.	PROGRAM APPLICA	TI	ONS	•				•	•		•				5
IV.	WIND RESULTS	•		•	•			•		•	•				б
v.	SUMMARY AND REC	COM	1ENI	AT I	ONS	3	•	•			•				11
	PERSONNEL LIST	ING					•				•	•		•	12
	ADDEMNIV														12

Adress1	on For	
Stis C	RARI	X
- 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	ent lon	
pictri	hnt*on/	
1	oblict <mark>v</mark> Salicy	
Dist		
A		

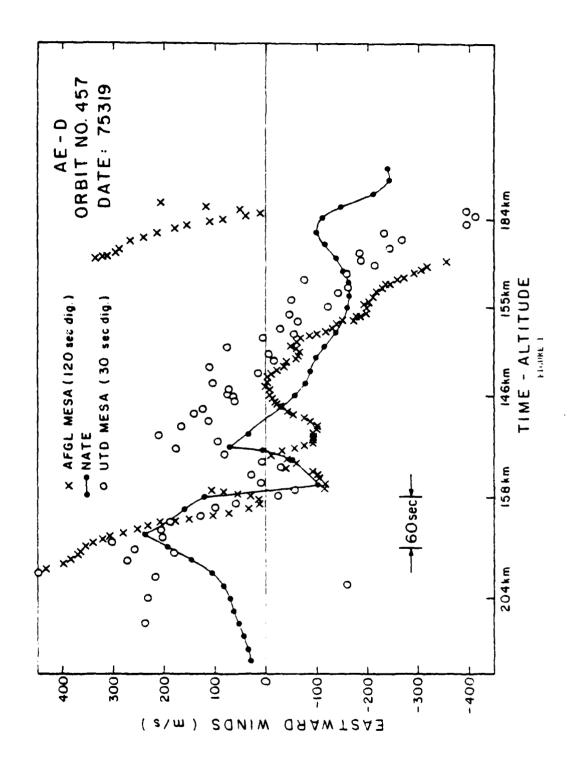
INTRODUCTION

The AFGL Miniature Electrostatic Accelerometer (MESA) on the Atmosphere Explorer (AE) satellites was designed to provide neutral atmospheric density measurements by determining satellite deceleration caused by aerodynamic drag. Each spacecraft contained three single-axis sonsors. The first is aligned with the Z (spin) axis, while the other two sensors (designated XY and YX) are located in the X-Y plane at a 45° angle on either side of the +X axis. The sensors have the capability of being commanded into any of three sensitivity ranges, with the least sensitive being only used for satellite thrust monitoring. The focus of our effort has been the utilization of the data from these three sensors to determine the velocity of the component of neutral atmospheric winds lying along the spacecraft Z axis during those periods when spacecraft altitude was low enough (<180 km) for as curate measurements.

The first section of the report discusses the software package that was put together at UTD to calculate the winds from the MESA telemetry data. This is followed by a discussion of the application of this software to actual MESA data. Results are then plotted and compared with NATE data, and the outputs of different filters are examined. Finally, some recommendations for MESA data analysis are presented.

SOFTWARE

The first step was the examination of the results of the AFGL MESA Data Reduction System (DRG), which included a preliminary wind estimate. It was initially apparent that the contribution to the winds of atmospheric corotation (9000 m/sec) was not included. In addition, the computed values showed sudden large discontinuities (see Figure 1). The cause of these effects required futher study. Also it was desirable to investigate the effectiveness of the digital filter used to remove noise from the data. For these reasons it was decided to develop an independent program to malvae the MESA data and compute the neutral winds. The software package developed is described in the following paragraphy.



To simplify calculations and to achieve maximum accuracy it was decided to use only data from despun passes. In addition, the program does not require any input from the MESA data reduction file system developed at the AFGL, using only the TM data base and orbital and attitude information from the AE Central Computer Facility at Goddard Space Flight Center. The program converts the TM data to MESA raw data, applies a temperature correction to the data, utilizes a filter to eliminate noise due to the satellite's momentum wheel assembly, and finally calculates the velocity of the neutral wind along the Z-axis of the satellite, producing a line printer plot of Z axis wind versus G.M.T. The Z axis wind was calculated from the formula:

$$W_{Z} = |\overline{V}_{S}'| \frac{(a_{Z} + |\overline{a}_{+}| \cos \alpha)}{|\overline{a}_{+}|}$$

$$\tag{1}$$

where W_{2} is the Z axis wind, a_{Z} is the Z axis acceleration, \overline{V}_{S} ' is the satellite velocity in the earth's rotating frame of reference, \overline{a}_{+} is the total satellite acceleration, and α is the angle between the Z axis and the satellite velocity vector. Total satellite acceleration may be determined either by using the output from all three sensors or by utilizing only the XY and/or YX sensor and correcting for the angle between the sensor and the spacecraft velocity vector.

Control and Input Data

Basic program control is simple. The major required parameters are satellite identification, start and stop dates and times, sensor(s) to be used to compute total drag, reference frame for wind calculation (corotating or non-corotating), and number of data points used in the filter. The program then proceeds to extract from the AE data base all data necessary, including raw IM data, sensor temperature, range indication, overflow flags, and position, attitude and velocity information.

Temperature Correction and Filtering

Now PM data is first temperature corrected and then ted into a buffer from where it is then passed through a filter to remove momentum wheel and mutatical noise. One of the filters used is the "15-10" filter developed at AFGL for use on the MESA system.

#Sec Appendix.

Wind Determination

Data from above 280 km altitude is used to establish a zero acceleration baseline for sensor response. Then using the XY sensor, the YX sensor, or all three sensors, the total drag induced deceleration of the vehicle is determined. Using the known attitude and velocity of the satellite, Equation 1 is then applied to yield the Z axis wind.

Overranging

When one of the sensors is in its most sensitive range it can happen that the acceleration along that axis may exceed the maximum $(2 \times 10^{-5} \text{ g})$ for that range. When this occurs an overrange flag is signaled in the telemetry for that sensor. When the XY or YX sensor is being used to determine total drag, an overrange condition causes that data point to be dropped. In intervals where the Z axis sensor overranges no useful wind information can be obtained from the present software, but it is likely that data could be recovered in these regions with a little more effort. This ususally occurs only below 140 km altitude.

Output

The program output consists of four data files. The first consists of processed analog data. The second contains processed digital data (this is the main output file). The third contains a line printer plot of the Z axis wind. The last is a file indicating data dropped due to errors or over-ranging.

PROGRAM APPLICATIONS

Since there is no existing wind data base from NATE for AE-C this satellite performance is most critical from an overall AE-science consideration. In the course of data reduction several constraints upon the usage of the Winds program in regard to AE-C data become apparent.

One problem exists regarding the calibration of the XY and YX sensors. If during the course of a low altitude pass one uses the acceleration computed from one of these sensors and the orientation angle of that lensor relative to the direction of satellite velocity one can obtain a value for the total drag induced acceleration of the vehicle. If this is done for

each of the two spin plane sensors one consistently finds that they are in disagreement, with the XY sensor giving values approximately 10 to 20% larger than those from the YX sensor. This seems to indicate the need for an in-flight calibration, and this can perhaps be achieved by utilizing the density measurements from the onboard neutral mass spectrometers.

A second concern is that of the sensitivity ranges of the sensors. The standard mode of operation appears to have been to leave the YX and Z sensors in their most sensitive range (range 3) while leaving the XY sensor in range 2. While this practice yields the maximum amount of information in regard to total atmospheric density, it requires that the XY and YX sensors be intercalibrated to produce a smooth wind profile, since each is used for part of each pass. The YX sensor saturates and overranges below 150 km altitude, forcing us to use only the XY sensor for total drag determination below this altitude.

In the case of the AE-D and AE-E satellites it was found that the XY and YX were also left on ranges 2 and 3 respectively. The Z sensor, however, was not set on the most sensitive range but instead on range 2. This degrades the accuracy of wind determination using these two spacecraft, but not by a large factor, because the signal to noise level is fairly low so that the extra gain $(x \ 20)$ going from range 2 to range 3 does not help too much (i.e. the signal is large enough to be seen on range 2.)

WIND RESULTS

Because of the short time available to examine MESA wind data after the new oftware was checked out the conclusions reached here are somewhat tentative. A total of approximately 40 orbits from all three satellites provide the basis for this report.

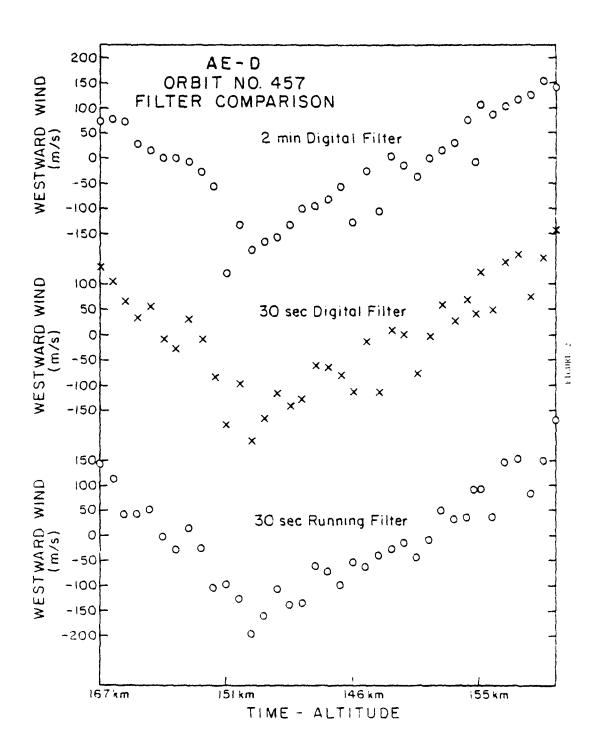
The AE-D data plotted in Figure 1 show a comparison of the east-west wind component as determined by the program developed here, by the AFGL program, and by the NATE instrument. Except for the regions where the AFGL result has an apparent sign difficulty, the three sets of data are in general agreement. The smoother appearance of the AFGL output compared to the UTD points arises because of much heavier filtering (a two-minute filter compared to a 30-second filter). Some of the AE-D MESA data is quieter than that shown here, and some is noisier, but there is much useful data even though the Z axis accelerometer was on the middle sensitivity range.

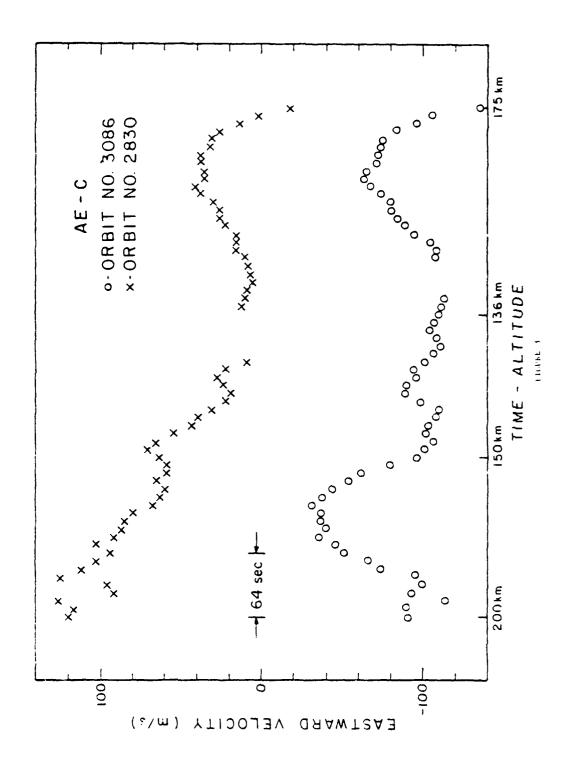
Perigee was near 82° north latitude on orbit 0-457 so that the wind pattern may be reasonable in spite of its structured appearance.

A more detailed comparison of the various filter outputs is shown in Figure 2, which also uses data for orbit D-457 (note the change in sign of the wind coordinate). Data points are plotted every 10 seconds. All the curves have pretty much the same general characteristics except for the slightly higher noise level with the narrower filters. The simple running filter seems to be as effective as the much more sophisticated digital filter.

There is no wind data with which to compare on AE-C, but the results obtained from the sensors using current software are consistent with those expected on a theoretical basis. Figure 3 presents two typical equatorial passes recorded nineteen days apart on the AE-C satellite. Note that in both cases there exist data gaps near perigee due to the saturation of the Z sensor (not all points are saturated, only those with large noise spikes). In addition, the effects of atmospheric corotation have been removed, corotation introducing an easterly bias of approximately 450 m/sec near the equator. The perigees of orbits 2830 and 3086 occur at a Solar Local Time (SLT) of about 16:40 and 12:00 respectively. Models of the wind velocity determined from atmospheric pressure gradients predict zonal wind velocities of about 100-150 m/sec in these regions, with the wind at 12:00 SL3 being westward and the wind at 16:00 SLT being eastward. Indeed, the data do reveal a westward to eastward shift in the wind velocity near peripee with the magnitude of the change being about 110 m/sec. The noise level of the rest of the AE-C data examined is of comparable magnitude. In general, this noise level is probably smaller than the absolute uncertainty in the winds arising from errors in the spacecraft attitude. The results certainly appear to be of useful quality for AE-science purposes.

A comparison of the NATE wind data from AE-E with that from MISA is shown in Figure 4 for orbit 385. There is a general consistency between the two results though there is an offset of 40 to 50 m/s near perigee. This discrepancy is not a great deal greater than the uncertainty in the mounting position of the devices within the spacecraft. From a comparison of a large data base of NATE winds with the ion drift meter results from AE-E, we also find an offset between the devices of the order of 50 m/s. This offset is slowly varying in time tweeks to months) and is not yet understood.





SUMMARY AND RECOMMENDATIONS

In summary, the technique of using accelerometers on a low altitude satellite to determine the transverse neutral wind appears to provide valid measurements. Though a relatively high noise level is present on the AE satellites, the current MESA sensors are adequate to provide a reasonable wind determination, particularly when the Z axis sensor is in its most sensitive range, as in the case of AE-C. AE-D and AE-E orbits, which have a slightly higher noise level, are nevertheless valuable in that NATE wind data provides a comparison between different techniques and can be used to build confidence in the AE-C orbits. This data should be completely analyzed and the results placed in the Unified Abstract (UA) files for reference by all interested researchers. A word assignment for MESA wind already exists in the UA files.

We did not expend any effort to examine the usefulness of the spinning data, but it is probably worth some effort to establish whether this data is of any value also.

It may be that the MESA data from both AE-D and AE-E should also be completely analyzed and placed in the UA files. Certainly a relatively thorough comparison with the NATE data should be made, if for no other reason than to determine the proper confidence to be placed in the technique for the AE-C wind users. This is probably a matter that might be discussed with the AE-science team.

At UTD we have developed a data analysis program monitor in connection with the processing of the RPA and MIMS data from AE. If this scheme were implemented for MESA, the further effort involved to complete the data analysis, place the data in the UA file, and provide a tilm plot of each orbit processed would be minimal. The program would have to run in the MESA Account. If this plan is to be given consideration, it should be implemented soon, because of the impending loss of the .9 computer for AE data processing.

PERSONNEL

A list of scientists, engineers, technicians, and programmers who contributed to the work is given below:

- W.B. Hanson, Principal Investigator
- W.R. Coley, Co-Investigator
- R.A. Heelis, Co-Investigator
- R.A. Power, Programmer
- J.K. Lowell, Programmer
- C.R. Lippincott, Project Manager

There were no previous or related contracts or previously produced publications or articles resulting from total or partial sponsorship of this contract.

APPENDIX

Neutral Wind Measurement

We desire to use the measured drag force \overline{F} on a satellite to determine the neutral gas winds (\overline{W}) . We can write (in the satellite trame):

$$\overline{V}_{n} = -\overline{V}_{S} + \overline{\omega} \times \overline{R} + \overline{W}$$
(1)

where \overline{V}_n = total neutral gas velocity in the satellite frame

 \overline{V}_{s} = satellite velocity

 $\frac{-}{\omega}$ = earth's angular velocity

 \overline{R} = satellite position vector

 \overline{W} = neutral wind in earth's rotating frame of reference.

The drag force on the satellite is:

$$\overline{F} = 1/2 \rho A C_D | \overline{V}_n | \overline{V}_n$$
 (2)

where ρ = atmospheric density

 $C_{\overline{D}}$ = drag coefficient in direction of \overline{V}_{n}

A = presentation area normal to \overline{V}_n of satellite.

Assume the satellite is oriented with its \hat{X} axis approximately in the ram direction and with its \hat{Z} axis horizontal. Then the acceleration components are:

$$a_{\overline{z}} = \overline{F} + \overline{Z}/M \tag{3}$$

$$\mathbf{a}_{\mathbf{X}} = \mathbf{F} \cdot \mathbf{\bar{X}}/\mathbf{M} \tag{4}$$

where M = satellite mass.

We also note that \hat{V}_n is parallel to \hat{a}_+ where $\hat{a}_+ = \hat{a}_x \hat{X} + \hat{a}_y \hat{Y} + \hat{a}_z \hat{Z}$.

Making the substitution $\overline{V}_{S}^{'} = \overline{V}_{S} - \overline{\omega} \times \overline{R}$ we can rearrange (1):

$$\overline{W} = \overline{V}_n + \overline{V}_s$$
 (5)

or looking at just the Z component

$$W_z = V_{nz} + V_{sz}$$
 (6)

Using eq. (3) and (4) we can show

$$\frac{a}{a}z = \frac{v_{nz}}{v_{nx}}$$
 (7)

Using the definition of V_{nx} :

$$V_{nx} = V_{n} + \hat{X}$$

$$= \sqrt{V_{n}} \cdot \frac{\tilde{V}_{+}}{\sqrt{\tilde{A}_{+}}} + \hat{X}$$

$$= \sqrt{V_{n}} \cdot \frac{\tilde{A}_{+}}{\sqrt{\tilde{A}_{+}}} + \hat{X}$$

$$= \sqrt{V_{n}} \cdot \frac{\tilde{A}_{+}}{\sqrt{\tilde{A}_{+}}}$$
(8)

Using eq. (2) to eliminate $|V_n|$ in eq. (8) we can then substitute this expression for V_{nx} into eq. (7) and get:

$$V_{\rm nz} = a_{\rm z} - (\frac{2M}{\rho A C_{\rm D} |\bar{a}_{+}|})^{1/2}$$
 (9)

Thus the expression for the Z component of the neutral wind becomes:

$$W_{2} = 1 \left(\frac{2M}{iM \cdot C_{D} \cdot (Y_{+})} \right)^{1/2} + V_{D}^{-1}$$
 (10)

 $V_{s'z}$ depends only on satellite position, velocity and the earth's rotation. These parameters can be obtained from the orbital-attitude data of the A.E. computer facility. However, an independent value of ρ is also required.

Alternatively, using equations (6), (7), and (8) we can eliminate the dependence of W_2 on the magnitude of ρ or C_D , and obtain:

$$W_{z} = \left| \overline{V}_{n} \right| \frac{a_{z}}{\left| \overline{a}_{+} \right|} + V_{SZ}$$
 (11)

Thus the fact that $C_{\overline{D}}$ and A depend on the angle of attack is cancelled out if Equation (11) is used. But even if Equation (10) is used, it is not likely that $C_{\overline{D}}$ will change very greatly over the small angles of attack normally encountered (<10°).

Making the approximation $|\tilde{V}_n| = |\tilde{V}_s|$ Equation (11) becomes:

$$W_{z} \doteq |\overline{V}_{s}'| \frac{a_{z}}{|\overline{a}_{+}|} + \overline{V}_{s}' \cdot \hat{Z}$$
 (12)

or

$$W_{z} \doteq |\overline{V}_{s}'| \frac{(a_{z} + |\overline{a}_{+}| \cos \alpha)}{|\overline{a}_{+}|} \equiv W_{z}'$$
 (13)

where α = angle between \overline{V}_{s} ' and \hat{z} .

Equation (13) is the same as Equation (!) in the text, and is used in the UTD winds program to determine W_Z . Without an accurate independent measurement of ρ no determination can be made of the other components of \widetilde{W} .

Estimate of Error. The validity of the approximation $|\vec{v}_n| = |\vec{v}_s|$ may be examined under typical conditions.

Using (11) and (13)

$$W_{z} = W_{z}^{+} + (|\vec{V}_{n}| - |\vec{V}_{n}|) + \frac{a_{z}}{|a_{+}|}$$
 (14)

we know $(|\overline{V}_n| - |\overline{V}_s|) \le |\widetilde{W}|$

and typically $a_z/\frac{1}{a_z} + \leq 0.08$

So for a 200 m/s wind the error term in (14) is \leq 16 m/s.

A secondary source of error is introduced in that $|\tilde{a}_{+}|$ is typically determined by using only one sensor and making the assumption that the neutral wind velocity is anti-parallel to the satellite velocity, thus we can write:

$$|\overline{a}_{+}| = a_{x} / (\widehat{x} \cdot \frac{\overline{v}_{n}}{|v_{n}|})$$

$$\stackrel{=}{=} a_{x} / (\widehat{x} \cdot \frac{\overline{v}_{s}'}{|v_{s}'|})$$
(15)

For $|\overline{W}| = 200$ m/sec this leads to a maximum 2.7% error in $|\overline{a}_{+}|$. Using (12) this implies an error ≤ 16 m/s. Thus, these two sources of error are about equal in magnitude.

